Matrix models in Chern-Simons theory

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  - Brief introduction to Chern-Simons (CS) theory

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- Strongly correlated models with topological properties
- Conclusions and outlook

- Let $H = (H_{jk})_{j,k=1}^{N}$ be a square $N \times N$ matrix with randomly distributed elements H_{jk}. This is a random matrix with respect to a probability distribution, defined by:

$$P_{\beta}^{(N)}(H) \propto \exp\left(-\beta \mathrm{TrV}(H)\right),$$

- The first and most studied ensembles are the Gaussian ensembles, $V(H) = H^2$. It can be actually shown that the previous expression is automatically restricted to the form

$$P(H) = \exp\left(-a\mathrm{Tr}H^2 + b\mathrm{Tr}H + c\right), \qquad a > 0,$$

  if one postulates statistical independence of the matrix elements H_{ij}. There are three different ensembles defined depending on the values of the parameter $\beta = 1, 2$ or 4.

Ensembles of random $N \times N$ matrices H are defined by the following demands:

1. The probability $P(H)d\left[H\right]$ is invariant under any transformation $H \to U^{-1}HU$, where U is either an orthogonal ($\beta = 1$), unitary ($\beta = 2$) or symplectic ($\beta = 4$) matrix. That is to say, if $H' = U^{-1}HU$ where U belongs to the unitary group $U(N;\beta)$, then $P(H')d\left[H'\right] = P(H)d\left[H\right].$

2. The matrix elements which are not related by the symmetry of the matrix are statistically independent (Gaussian ensembles)

- Diagonalization: for each matrix H there is a matrix U that maps it onto its eigenvalues. The Jacobian of the transformation is $J_\beta(\{x_i\}) = \prod_{i<j} |x_i - x_j|^\beta$. The resulting expression with a generic potential is:

$$P(x_1, ..., x_N) = \mathrm{C_N} \prod_{i<j} |x_i - x_j|^\beta \prod_{i=1}^{N} \mathrm{e}^{-\frac{\beta}{2} V(x_i)}.$$

  The potential $V(x) = \log^2 x$ (log-normal weight function $\omega(x) = \mathrm{e}^{-\log^2 x}$) is at the center of most developments in this thesis.
- The main relevant quantities are m-partial integrations over the previous N-dimensional probability density function

- A central and powerful result in random matrix theory is that m-point correlation function can be computed from the two-point kernel as follows (simplest case of an Hermitian ($\beta = 2$) ensemble)

$$R_m^{(N)}(x_1, ..., x_m) = \det \left(K_N \left(x_i, x_j \right) \right)_{1 \leq i,j \leq m}$$

- Orthogonal polynomials method $\Longrightarrow$ explicit expressions for $K_N \left(x_i, x_j \right)$. Let $p_N(x) = c_N x^N + ...$ the *Nth* orthogonal polynomial associated to $\mathrm{e}^{-V(x)}$, the two-point kernel is

$$\begin{aligned} K_N \left(x, y \right) &= \mathrm{e}^{-\frac{V(x)+V(y)}{2}} \sum_{i=0}^{N-1} p_i \left(x \right) p_i \left(y \right) \\ &= \frac{c_{N-1}}{c_N} \frac{p_N(x) p_{N-1}(y) - p_{N-1}(x) p_N(y)}{x - y} \mathrm{e}^{-\frac{V(x)+V(y)}{2}} \end{aligned}$$

- We consider Chern-Simons theory on a three-manifold M and for a simply-laced gauge group G, with action

$$S(A) = \frac{k}{4\pi}\int_M \mathrm{Tr}\left(A \wedge dA + \frac{2}{3}A \wedge A \wedge A\right),$$

where A is a connection on M.

- Witten showed in 1989, that the partition function of Chern-Simons theory

$$Z_k(M) = \int \mathcal{D}A e^{iS_{\mathrm{CS}}(A)},$$

defines a topological invariant .

- The contribution of reducible flat connections to the Chern-Simons partition function of $X(\frac{p_1}{q_1}, \cdots, \frac{p_n}{q_n})$ for $U(N)$ is given by (M. Mariño, Comm. Math. Phys. 253, 25 (2004))

$$Z_{\rm CS}(M) = \int_{-\infty}^{\infty} \prod_{i=1}^{N} {\rm d}y_i \, {\rm e}^{-y_i^2/2g_s - l t_i y_i} \frac{\prod_{j=1}^{n} \prod_{k<l} 2\sinh \frac{y_k - y_l}{2p_j}}{\prod_{k<l} \left(2\sinh \frac{y_k - y_l}{2}\right)^{n-2}}$$

- The simplest case is S^3 and gauge group $U(N)$

$$Z_{\rm CS}(S^3) = \left(\frac{g_s}{2\pi}\right)^{-\frac{N}{2}} \int_{-\infty}^{\infty} \prod_i \frac{du_i}{2\pi} e^{-\frac{u_i^2}{2g_s}} \prod_{i<j} \left(2\sinh\left(\frac{u_i - u_j}{2}\right)\right)^2$$

- Thus, Mariño ended up with N-dimensional integral expressions for Chern-Simons partition functions whose expression resemble that of random matrix theory.

- Ingredients: 1) Change of variables $e^{u_i} = x_i$ 2) Symmetry of the log-normal $\omega(xq) = \sqrt{q}x\omega(x)$ (when $\omega(x) = e^{-\log^2 x_i/2g_s}$), then

$$Z\left(S^3\right) = \int \prod_{i=1}^{N} \frac{du_i}{2\pi} e^{-\frac{u_i^2}{2g_s}} \prod_{i<j} \left(2\sinh\left(\frac{u_i - u_j}{2}\right)\right)^2$$

$$= \quad (2\pi)^{-N} e^{\frac{-N^3 g_s}{2}} \int \prod_{i=1}^{N} dx_i e^{-\frac{\log^2(x_i)}{2g_s}} \prod_{i<j} (x_i - x_j)^2 .$$

- Last expression is the Stieltjes-Wigert matrix model. For the partition function computation, we actually only need the leading coefficients, $p_i(x) = a_i x^i + \ldots$ which are

$$a_j = q^{(j+1/2)^2} \left\{(1-q)\ldots\left(1-q^j\right)\right\}^{-1/2} .$$

Chern-Simons theory and the Stieltjes-Wigert matrix model

Partition function (M.T., Mod.Phys.Lett. A19, 1365 (2004))

- The partition function in terms of the orthogonal polynomials is:

$$
\begin{aligned}
Z &= \int \ldots \int \prod_{i=1}^{N} \omega\left(x_i\right) dx_i \prod_{i<l} \left(x_i - x_l\right)^2 \\
&= \frac{N!}{\prod_{i=0}^{N-1} a_i^2} = N! a_0^{-2N} \prod_{i=1}^{N-1} \left(\left(\frac{a_{i-1}}{a_i} \right)^2 \right)^{N-i} .
\end{aligned}
$$

- Using the coefficients, we have $\left(\frac{a_{j-1}}{a_j}\right)^2 = q^{-4j}\left(1-q^j\right)$ and $a_0 = q^{1/4}$, leads to

$$
Z_{\mathrm{SW}} = N! q^{-\frac{1}{6}N(2N-1)(2N+1)} \prod_{j=1}^{N-1} \left(1-q^j\right)^{N-j}
$$

- Then, the N-dimensional integral is computed explicitly:

$$Z_{\sinh} = \left(\frac{g_s}{2\pi}\right)^{N/2} N! \mathrm{e}^{\frac{1}{6}g_s N\left(N^2-1\right)} \prod_{j=1}^{N-1} \left(1-q^j\right)^{N-j},$$

and transforming the product term and identifying $g_s = \frac{2\pi i}{k+N}$ (coupling constant with CS parameter) we finally find:

$$Z\left(S^3\right) = \mathrm{e}^{\frac{1}{4}i\pi N^2} \left(k+N\right)^{-N/2} \prod_{j=1}^{N-1} \left(2\sin\frac{\pi j}{k+N}\right)^{N-j}.$$

- We also have studied the biorthogonal ensemble

$$
\begin{aligned}
Z^{P,Q} &= \int \prod_i \frac{du_i}{2\pi} e^{-u_i^2/2g_s} \prod_{i<j} (2\sinh(\frac{u_i - u_j}{2P}))(2\sinh(\frac{u_i - u_j}{2Q})) \\
&= q^{-\frac{N\alpha^2}{2}} \int \prod_i \frac{dy_i}{2\pi} e^{-\kappa^2 \log^2 y_i} \prod_{i<j} (y_i^{1/P} - y_j^{1/P})(y_i^{1/Q} - y_j^{1/Q}),
\end{aligned}
$$

- For this we had to find out the biorthogonal version of the Stieltjes-Wigert polynomials

$$
\begin{aligned}
\int Y_n(x,k)\, x^{kj} \omega(x)\, dx &= \alpha_n^{(k)} \delta_{n,j}, \\
\int Z_n(x,k)\, x^{j} \omega(x)\, dx &= \beta_n^{(k)} \delta_{n,j}.
\end{aligned}
$$

not available in the literature.

- More analytical computations done. The Chern-Simons invariant of the unknot are quantum dimensions. We showed

$$\begin{aligned} < \ \mathfrak{s}_\lambda(M) >_w &= \int [dM] \mathfrak{s}_\lambda(M) \mathrm{e}^{-\frac{1}{2g_s}\mathrm{Tr}(\log M)^2} \\ &= q^{-n|\lambda| - \frac{1}{2} C_\lambda^{U(n)}} \mathcal{D}_\lambda . \end{aligned}$$

where $C_\lambda^{U(n)}$ is the Casimir of $U(N)$ and the last term are the quantum dimensions:

$$\mathcal{D}_\lambda \equiv \prod_{x \in \lambda} \frac{\lfloor n + c(x) \rfloor}{\lfloor h(x) \rfloor},$$

where for each box of the diagram $h(x) \equiv \lambda_i + \lambda_j' - i - j + 1$ is the hook-length and $c(x) \equiv j - i$ the content of x.

- Let us introduce the topic with a simple example. Stieltjes computed (1894) with $\vartheta \in [-1, 1]$

$$f_{\vartheta}(x) = \mathrm{e}^{-\log^2 x}\left(1 + \vartheta \sin\left(2\pi \log x\right)\right),$$

$$\frac{1}{\sqrt{\pi}} \int_0^{\infty} \mathrm{d}x\, x^n f_{\vartheta}(x) = \mathrm{e}^{(n+1)^2/4}.$$

- Thus, the parameter ϑ does not play any role regarding integer moments and the infinitely many functions in the family $f_{\vartheta}(x)$ all have the same integer moments.

Moment problem and non-uniqueness

Two deformations of the log-normal (S.dH. and M.T,, Nucl.Phys. B731, 225 (2005))

$$f(x) = \mathrm{e}^{-k^2 \log^2 x} \left(1 + \lambda \sum_{n=1}^{m} a_n \sin\left(2\pi b_n \frac{\log x}{\log q} \right) \right)$$

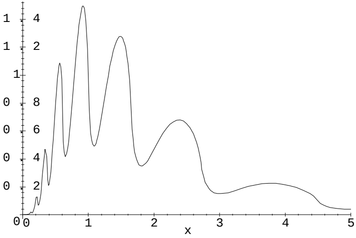

Figure: $a_n = n^{-2}, b_n = n, k = 1, \lambda = 0.5, m = 200$

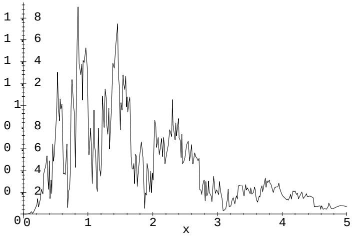

Figure: $a_n = n^{-1}, b_n = n^2, k = 1, \lambda = 0.5, m = 50$

Moment problem and non-uniqueness

Summary of relevant properties (S.dH. and M.T., Nucl.Phys. B731, 225 (2005))

- Infinitely many weight functions with the same moments and hence the same orthogonal polynomials. Thus, there are infinitely many matrix models with the same properties (e.g. same CS partition function).
- One can extend the Stieltjes result and construct a Fourier series of log-periodic oscillations.
- Using log-normal symmetry equation one can show that the multiplication of a function $g(x) = g(qx)$ (q-periodic) does not alter its moments. Thus, the log-normal can be locally deformed by a self-similar function, that can be fractal.
- Deformations are of log-periodic type, signature of a discrete scale invariance. Many other physical models are characterized by DSI (Choptuik's scaling, q-deformed models with q real, renormalization group limit cycles, ...)

Moment problem and non-uniqueness

Discretization (S.dH. and M.T., Nucl.Phys. B731, 225 (2005))

- Any undetermined moment problem admits a discrete solution. In the log-normal case, it is known explicitly (Chihara 1970 and 1979)

$$w(x) = \frac{1}{\sqrt{q}M(c)} \sum_{n=-\infty}^{\infty} c^n q^{\frac{n^2}{2}+n} \delta(x - cq^n)$$

- This leads to the equivalence of the continuous model with the discrete one

$$\left(\frac{g_s}{2\pi}\right)^{-\frac{N}{2}} \int \prod_i \frac{du_i}{2\pi} e^{-\frac{u_i^2}{2g_s}} \prod_{i<j} \left(2\sinh\left(\frac{u_i - u_j}{2}\right)\right)^2$$

$$= \quad A_N \sum_{n_1,\ldots,n_N=-\infty}^{\infty} e^{-\frac{g_s}{2}\sum_i n_i^2} \prod_{j<k} \left(2\sinh\left(\frac{g_s}{2}(n_j - n_k)\right)\right)^2$$

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  - Paul Levy (1939), K. Ito (1942/1946). More complete mathematical foundation. Ito calculus for stochastic processes.

- 1D Brownian motion: the transition probability density is Gaussian and satisfies Heat equation (it is a diffusion process):

$$p_t(x,y) = \frac{1}{\sqrt{2\pi D t}} \exp(-\left(x-y\right)^2/2Dt)$$

$$\frac{\partial p_t(x,y)}{\partial t} = D\frac{\partial^2 p_t(x,y)}{\partial x^2}$$

- Michael Fisher model of N vicious walkers (*Walks, Walls, and Melting, J. Stat. Phys. 34, 667, (1984)*):

$$p_{t,N}(\lambda,\mu) = \frac{1}{(2\pi t)^{N/2}} \mathrm{e}^{-\frac{|\lambda|^2+|\mu|^2}{2t}} \det\left|\mathrm{e}^{\lambda_i\mu_j/t}\right|_{1\leq i<j\leq N} .$$

- We consider now initial and final boundary conditions, $\mu = \lambda$, and an equal spacing condition, that is, $\lambda_{0j} - \lambda_{0,j+1} = a$, where a is the initial and final spacing between two neighboring movers and compute the probability of a reunion:

$$p_{t,N}(\lambda_0, \lambda_0) = \frac{1}{(2\pi t)^{N/2}} \prod_{k=1}^{N} (1 - \mathrm{e}^{-ka^2/t})^{N-k}$$

- If we choose $a^2 = 1$ and identify $-\frac{1}{t} = g_s = \frac{2\pi i}{k+N}$ then

$$Z_{\mathrm{CS}}(S^3) = \mathrm{e}^{\frac{\pi i}{2} N^2} q^{-\frac{1}{12} N(N^2-1)} \, p_{t,N}(\lambda_0, \lambda_0),$$

where the label 0 refers to the Weyl vector $\lambda_0 = \rho$

- By imposing the same initial and final conditions we are dealing with N non-intersecting Brownian motions on the surface of a cylinder. This geometry will appear in the next -and last- Section.
- Other invariants are obtained with other boundary conditions:

$$\begin{aligned} p_{t,r}(\lambda,\rho) &= \frac{1}{(2\pi t)^{r/2}}\,\mathrm{e}^{-\frac{|\lambda|^2+|\rho|^2}{2t}} \prod_{\alpha>0} 2\sinh\frac{(\alpha,\lambda)}{2t} \\ &= \left\langle W_{\lambda}\left(\mathrm{unknot}\right)\right\rangle , \end{aligned}$$

and $p_{t,r}(\lambda,\mu)$ corresponds to the Hopf link invariant $\left\langle W_{\mu\lambda}\left(\mathrm{Hopf}\right)\right\rangle$ (S.dH.).

- The Wigner-Dyson distribution of eigenvalues $P_N^\beta(\{x_i\})$ coincides with the probability distribution of the N-particle coordinates $P_N^\beta(\{r_i\})$ of the quantum ground state of a $1D$ Hamiltonian: the Calogero model:

$$\Psi_0(x_1, ..., x_N) = C_N \prod_{i<j} |x_i - x_j|^{\beta/2} \prod_{i=1}^{N} \mathrm{e}^{-\frac{\beta}{4}x_i^2},$$

$$H = -\sum_{i=1}^{N} \frac{d^2}{dx_i^2} + \beta\left(\frac{\beta}{2} - 1\right)\sum_{i<j}\frac{1}{(x_i - x_j)^2} + \frac{\beta^2}{4}\sum_{i=1}^{N} x_i^2.$$

- Is there an analogous result for the CS matrix model ? The literature (e.g. V. I. Inozemtsev and D.V. Mescheryakov, Phys. Lett. A, 106, 101 (1984)) seems to say no, but the answer is affirmative.

- The result is (counterexample by Forrester (1994) to Inozemtsev (1989)):

$$H = -\sum_{i=i}^{N} \frac{\partial^2}{\partial x_i^2} + \frac{1}{g_s^2} \sum_{i=1}^{N} x_i^2 + \frac{m(m-1)}{2L} \sum_{i<j} \frac{1}{\sinh^2\left(\frac{x_i - x_j}{2L}\right)} + \frac{m}{g_s L} \sum_{i<j} (x_i - x_j) \coth\left(\frac{x_i - x_j}{2L}\right),$$

- If $\Psi_0 = \mathrm{e}^{-\mathcal{H}}$ then $\mathcal{H} = \frac{1}{2g_s} \sum_{i=1}^{N} x_i^2 - \sum_{i<j} \ln \sinh \left| \frac{x_i - x_j}{2L} \right|$. The last term is the Coulomb potential between charges in the surface of a $2D$ cylinder

- We compute the pairs H, Ψ_0 for the orthogonal and symplectic groups case.
- We show that the CS exactly solvable model behaves as a system of N free fermions at finite temperature
  - The two-point kernel of the model is $K(x,y) = \frac{a\sin(\pi(x-y))}{\sinh(a\pi(x-y))}$ like free fermions at finite T. There is exponential decay of correlations: finite temperature diminishes correlation between particles.
  - The density matrix of a system of 1D free fermions at finite T and harmonic confinement coincides with the Brownian motion density

- Quantum topological invariants can be extracted from Ψ_0 in two ways:
  - As the norm of $|\Psi_0|^2$ as is typical of Chern-Simons theory $Z(S^3) = \langle \Psi_0 | \Psi_0 \rangle$, but with the N-body wavefunctions instead of wavefunctionals.
  - Specifying a very particular configuration where the fermions are equispaced
- Since the Gaussian factor in the CS wavefunction contributes a phase factor, the Sutherland model also leads to the CS partition function when evaluated in a crystalline configuration.

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  - The continuous deformations case is characterized by log-periodic oscillations due to the discrete scale symmetry of the log-normal weight
  - Discretization: the model can be discretized and the resulting N-dimensional sums are useful in the study of q-2D Yang-Mills

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- The appearance of random matrices indicates connections with condensed matter models. In particular:
  - Interpretation of the models as a model of Calogero type models with different and additional two-body interactions is possible
  - The models are many-body systems -with a Coulomb gas interpretation- with quantum topological properties

Open problems

- Further study of the models along a condensed matter line: 1D solvable models and their quantum topological properties, relationship with the charged Bose gas model and Laughlin wavefunction on cylindrical geometries.

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